BASIC CALCULUS

CHAPTER 1

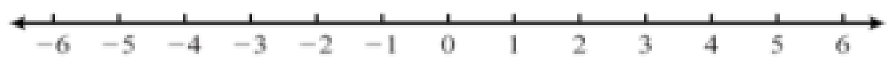
REVIEW OF THE REAL NUMBERS SYSTEM AS AN ORDERED FIELD

1.1REPRESENTATION OF THE REAL LINE

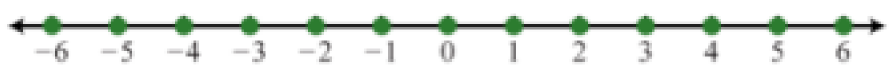
A real number line, often known as a number line, is a visual representation of real numbers that associates them with distinct points on a line. A coordinate is the real number that corresponds to a certain point. A graph is a point on the real number line that corresponds to a coordinate.

To create a number line, draw a horizontal line with arrows at both ends to indicate that it continues unbounded. Next, choose any location to symbolize the number zero; this point is known as the origin.

Mark consistent lengths on both sides of the origin, then indicate each tick mark to determine the scale. Positive real numbers lie to the right of the origin, while negative real numbers lie to the left. The number 0 is neither positive nor negative. Typically, each tick represents a single unit.



The graph of each real number is represented by a dot at the relevant place on the number line. The partial graph of the set of integers Z is as follows:



The real number system is made up of a set R of elements called real numbers and two operations called addition and multiplication, represented by the characters + and ·, respectively. If a and b are elements of the set R, a+b denotes the sum of a and b, whereas a·b (or ab) represents their product. The equation defines the operation of subtraction.

$$a - b = a + (-b)$$

where -b represents the negative of b, such that b(-b) =0. The operation division is determined by the equation

$$a \div b = a \cdot b^{-1} \quad b \neq 0$$

where b^{-1} represents the reciprocal of b, such that $b \cdot b^{-1} = 1$.

A set of axioms can fully characterize the real number system. Axiom refers to a formal assertion that is presumed to be true without proof. With these axioms, we may infer the properties of real numbers, which correspond to the familiar algebraic operations of addition, subtraction, multiplication, and division, as well as the algebraic notions of equation solving, factoring, and so on.

Theorems are properties that may be demonstrated to follow logically from axioms. Most theorem statements consist of two parts: the "if" part, known as the hypothesis, and the "then" part, known as the conclusion. A proof is the collection of arguments that verify a theorem. A proof is a demonstration that the conclusion follows from the assumption that the hypothesis is true.

A real number is either positive, negative, or zero, and any real number can be classified as either rational or irrational. A rational number is one that can be expressed as the ratio of two integers. That is, a rational number is of the form p/q, where p and q are integers and $q \neq 0$.

The rational numbers consist of the following:

The integers (positive, negative, and zero)

..., −5, −4, −3, −2, −1, 0, 1, 2, 3, 4 ,5, ...

The positive and negative fractions, such as

$$\frac{2}{7}, -\frac{3}{7}, \frac{67}{3}$$

The positive and negative terminating decimals, such as

$$2.36 = \frac{236}{100}, -0.003251 = -\frac{3{,}251}{1{,}000{,}000}$$

The positive and negative nonterminating repeating decimals, such as

$$0.333\ldots = \frac{1}{3}, -0.549549549\ldots = -\frac{61}{111}$$

The real numbers that are no rational are called irrational numbers. These are positive and negative nonterminating nonrepeating decimals, for example

$$\sqrt{3} = 1.732\ldots \quad \pi = 3.14159\ldots$$

A set is defined as a collection of items, with the objects in the set referred to as elements. If every element in a set S is also an element in another set T, then S is a subset of T.

We use the symbol ϵ to indicate that a specific element belongs to a set. Hence, we may write $8 \in N$, which is read as “8 is an element of N.” The notation $a, b \in S$ indicates that both a and b are elements of S. The symbol ∉ is read “is not an element of.” Thus, we read 1/2 ∉ N as “1/2 is not an element of N.”

A pair of braces {a} used with words or symbols can describe a set.

Method of Writing Set

A. Roster method. The set's items are listed and separated by commas; this is also known as the tabulation approach.

B. Rule Method. A description phrase, also known as set builder notation, is used to describe the elements or members of a set. The symbol is {x|P(x)}.

Example: If S is the set of natural number less than 6, we can write the set S as

{1, 2, 3, 4, 5}

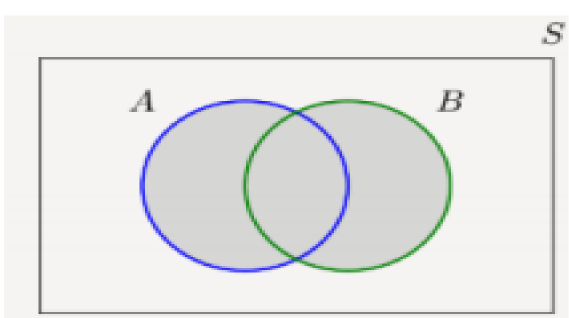
We can also write the set S as {x|x is a natural number less than 6}

LET’S RECALL ABOUT THE OPERATIONS ON SET

The union of A and B (A ∪ B) is the set of all items x in U that are either in A or B. Symbolically, A ∪ B equals {x|x ∈ A v x ∈ B}. Let A and B be subsets of the universal set U.

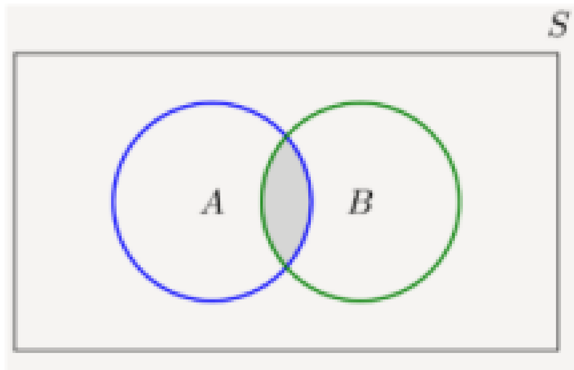
Example of Union Set

If A_1 = {a, b, c}, A_2 = {c, h}, A_3 = {a, d} then U_1A_1 = $A_1 \cup A_2 \cup A_3$ = {a, b, c, h, d}. We can similarly define the union of indefinitely many sets $A_1 \cup A_2 \cup A_3 \cup$ ...



Example of Intersection Set

If {1, 2} ∩ {2, 3} = {2} {1, 2} ∩ {2, 3} = {2}. In Figure below, the intersection of set A and B is shown by the shaded area using a Venn diagram.



1.2. INEQUALITIES AND INTERVALS

DEFINITION 1.2

If a, b ∈ R,

(i) a ≤ b if and only if either a < b or d = b;

(ii) a ≥ b if and only if either a > b or a = b

Inequalities are defined as the statements a < b, a > b, a ≤ b, and a ≥ b. Strict inequalities are defined as a < b and a > b, while nonstrict inequalities are defined as a ≤ b or a ≥ b.

THEOREM 1.2.1

(i) If a > 0 and b > 0, then a + b > 0.

(ii) If a > 0 and b > 0, then ab > 0.

THEOREM 1.2.2

(i) If a > 0 if and only if a is positive;

(ii) If a < 0 if and only if a is negative.

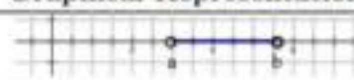
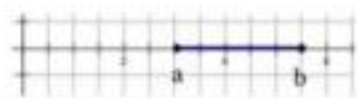
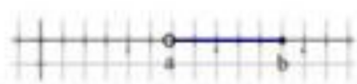
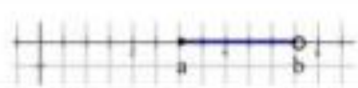
THEOREM 1.2.3

> Transitive Property of order
>
> If a, b, c, ∈ R, and If a < b and b < c, then a < c

THEOREM 1.2.4

> Suppose a, b, c, ∈ R
>
> (i) If a > b, then a + c > b + c.
>
> (ii) If a < b and c > 0, then ac < bc.
>
> (iii) If a < b and c < 0, then ac > bc.

Intervals is an important property of the real numbers is that they are totally ordered, so, we can compare any two real numbers, a and b, and make a statement of the form a≤b or b≤a, with strict inequality if a 6=b. Given any two points on the real line, a and b, we call the set of points between a and b an interval.

Notation	Set Description	Graphical Representation
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	
$(a, b]$	$\{x \mid a < x \leq b\}$	
$[a, b)$	$\{x \mid a \leq x < b\}$	
$[a, \infty)$	$\{x \mid a \leq x < \infty\}$	
(a, ∞)	$\{x \mid a < x < \infty\}$	
$(-\infty, b]$	$\{x \mid -\infty < x \leq b\}$	
$(-\infty, b)$	$\{x \mid -\infty < x < b\}$	
$(-\infty, \infty)$	$\{x \mid -\infty < x < \infty\}$	entire number line.

Example 1 Find and show on the real number line the solution set of the inequality

$$2 + 3x < 5x + 8$$

Solution: The following inequalities are equivalent

$$2 + 3x < 5x + 8$$

$$2 + 3x - 2 < 5x + 8 - 2$$

$$3x < 5x + 6$$

$$-2x < 6$$

$$x > -3$$

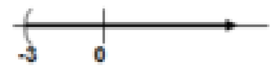


Figure 1

Therefore, the solution set is the interval (-3, +∞), which is illustrated in Figure 1.

Example 2 Find and show on the real number line the solution set of the inequality

$$4 < 3x - 2 \leq 10$$

Solution: By adding 2 to each member of the given inequality we obtain the equivalent inequalities

$$6 < 3x \leq 12$$

$$2 < x \leq 4$$

Figure 2

Thus, the solution set is the interval (2,4] as shown in Figure 2.

Example 3

Find and show on the real number line the solution set of the inequality $\frac{7}{x} > 2$

Solution: Multiply both sides of the given inequality by x, we obtain the following equivalent

inequalities:

$$7 > 2x$$

$$\frac{7}{2} > x$$

$$x < \frac{7}{2}$$

Therefore, the solution set is $\{x|x > 0\} \cap \{x|x < \frac{7}{2}\}$ or, equivalent, the set $\{x|0 < x < 7/2\}$, which is the interval $(0,7/2)$ shown in Figure 3.

1.3. ABSOLUTE VALUES

The concept of a number's absolute value appears in several important definitions. You will also need to deal with inequalities affecting absolute value.

DEFINITION 1.3.1

> The absolute value of x, denoted by |x|, is defined by
>
> $$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

DEFINITION 1.3.2

$$\boxed{|x| < a \leftrightarrow -a < x < a \text{ where } a > 0}$$

DEFINITION 1.3.3

$$\boxed{|x| < a \leftrightarrow x > a \text{ or } x < -a \text{ where } a > 0}$$

Example 1

Solve each of the equations for given the $|3x+2|=5$

Solution: This equation will be satisfied if either

$$3x + 2 = 5 \qquad \text{or} \qquad -(3x + 2) = 5$$

$$x = 1 \qquad\qquad\qquad x = -\frac{7}{3}$$

Example 2

Find and show on the real number line the solution set of the inequality $|x - 5| < 4$

$$|x - 5| < 4$$

$$-4 < x - 5 < 4$$

$$1 < x < 9$$

Therefore, the solution set is the open interval (1,9).

Example 3

Find the solution set of the inequality $|3x+2|>5$

Solution: The given inequality is equivalent to $3x + 2 > 5$ or $3x + 2 < -5$

$$3x + 2 > 5$$

$$x > 1$$

Therefore, in every number in the interval $(1, +\infty)$ is a solution.

$$3x + 2 < -5$$

$$x < -\frac{7}{3}$$

Hence, every number in the interval $(-\infty, -\frac{7}{3})$ is a solution.

The solution set of the given inequality is therefore $(-\infty, -\frac{7}{3}) \cup (1, +\infty)$.

REFERENCES

Winston S. Sirug. (2014). College Algebra. Revised Edition.

Louis Leithold. The Calculus With Analytic Geometry. Sixth Edition.

For further understanding of this LESSON visit these links

https://youtu.be/o08Ry0gqeKM

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